Charged-Particle Stopping Powers in Inertial Confinement Fusion Plasmas

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The effects of large-angle scattering, potentially important for plasmas for which the Coulomb logarithm is of order 1, have been properly treated in calculating the range (R) and the ρR (the fuel-areal density) of inertial confinement fusion plasmas. This new calculation, which also includes the important effects of plasma ion stopping, collective plasma oscillations, and quantum effects, leads to an accurate estimate, not just an upper limit of ρR. For example, 3.5 MeV α's from D-T fusion reactions are found to directly deposit $\simeq$47% of their energy into 20 keV deuterons and tritons. Consequently the α range (R) and ρR are reduced by about 60% from the case of pure electron stopping.

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The stopping of charged particles (α's, ^{3}H, ^{3}He, hot electrons,...) in compressed pellet plasmas is a fundamental problem with close parallels to early work of Rutherford and Bohr who studied α stopping in solid materials [1]. In the context of inertially confined fusion plasmas (ICF), it involves the deposition of energy from charged particles, especially the α's, in the fuel material during the initial cold and compressed state and then during the evolution to full ignition and burn [2–4]. The tremendous range of pellet plasma conditions [$n_e \lesssim 10^{27}$ cm^{-3} and $0.1 \lesssim T_e(T_i) \lesssim 40$ keV] is directly reflected in the range of the Coulomb logarithm—$1 \lesssim \ln\Lambda_b \lesssim 12$—a parameter fundamental to many plasma properties [5–12], including charged-particle stopping [2–4,13,14]. In the context of solutions to the plasma Fokker-Planck equation, $\ln\Lambda_b$ has a precise significance: It is a measure of the importance of small-angle collisions to large-angle scattering. Previously practical results based on the plasma Fokker-Planck equation have been well approximated only for $\ln\Lambda_b \gtrsim 10$ [5–12] because terms of $1/\ln\Lambda_b$ are truncated in the collision operator. Although this issue has remained unsolved until now, Fraley *et al.* [14] and later Mehlhorn [13] clearly recognized its relevance in their studies of α energy deposition in ICF. These workers noted that when ion stopping is significant for α's, large-angle scattering is also likely to be important. Fraley *et al.* did calculate ion stopping, but because they were unable to estimate the effect of the large-angle scattering within the framework of the Fokker-Planck approximation, and because they neglected collective plasma effects, they concluded that their estimate of range (R) and ρR (fuel-areal density) was in fact only an upper limit. Mehlhorn attempted to treat large-angle scattering [13], but discarded his results in favor of the standard small-angle formulation when the large-angle results proved inconsistent. In addition, Fraley *et al.* and many others have applied stopping power formulas to $\ln\Lambda_b \gtrsim 2$ plasmas in spite of the fact that these binary collision formulas were derived under the assumption that $\ln\Lambda_b \gtrsim 10$. Therefore there was inadequate justification in such applications.

These particular problems are overcome by our recent generalization of the Fokker-Planck equation, which properly treats the effects of large-angle scattering as well as small-angle collisions [15], and which is justified for application to $\ln\Lambda_b \gtrsim 2$ plasmas. In subsequent discussion of charged-particle stopping, we utilize one of the general results of that analysis,

$$\frac{dE^{t/f}}{dx} = -\frac{(Z_t e)^2}{v_t^2}\omega_{pf}^2 G(x^{t/f})\ln\Lambda_b\,, \tag{1}$$

where $dE^{t/f}/dx$ is the stopping power of a test particle (subscript or superscript t) in a field of background charges (subscript or superscript f) and

$$G(x^{t/f}) = \mu(x^{t/f}) - \frac{m_f}{m_t}\left\{\frac{d\mu(x^{t/f})}{dx^{t/f}} - \frac{1}{\ln\Lambda_b}\left[\mu(x^{t/f}) + \frac{d\mu(x^{t/f})}{dx^{t/f}}\right]\right\}. \tag{2}$$

The contribution of large-angle scattering is solely manifested by $1/\ln\Lambda_b$ terms of Eq. (2). In particular, if $\ln\Lambda_b \gtrsim 10$ and we ignore this correction, then Eqs. (1) and (2) reduce to Trubnikov's expression [8]. In the above equations, $Z_t e$ is the test charge; v_t (v_f) is the test (field) particle velocity with $x^{t/f} = v_t^2/v_f^2$ ($v_f^2 = 2kT_f/m_f$); m_t (m_f) is the test (field) particle mass; $\omega_{pf} = (4\pi n_f e_f^2/m_f)^{1/2}$, the field plasma frequency; $\mu(x^{t/f}) = 2\int_0^{x^{t/f}} e^{-\xi}\sqrt{\xi}\,d\xi/\sqrt{\pi}$ is the Maxwell integral; $\ln\Lambda_b = \ln(\lambda_D/p_{\min})$, where, for the nondegenerate regime, λ_D is the Debye length and $p_{\min} = [p_\perp^2 + (\hbar/2m_r u)^2]^{1/2}$; $p_\perp = e_t e_f/m_r u^2$ is the classical impact parameter for 90° scattering, with m_r the reduced mass and u the relative velocity. However, in the low-temperature, high-density regime, electron (not ion) quantum degeneracy effects must be considered in calculating λ_D and $p_{\min}$ [see Fig. 1(a)].

In addition to stopping by binary collisions, both small- and large-angle, stopping occurs due to plasma oscillations [16,17]. As this contribution is only important when $x^{t/f} \gg 1$, a generalized stopping formula is

$$\frac{dE^{t/f}}{dx} \simeq -\frac{(Z_t e)^2}{v_t^2}\omega_{pf}^2 \{G(x^{t/f})\ln\Lambda_b + \Theta(x^{t/f})\ln[1.123(x^{t/f})^{1/2}]\}. \quad (3)$$

Collective effects are represented by the second term $\{\ln\Lambda_c^{t/f} \equiv \ln[1.123(x^{t/f})^{1/2}]\}$ where $\Theta(x^{t/f})$ is a step function whose value is identically 0 (1) for $x^{t/f} \leq 1$ (>1). Note that for all charged-fusion products (α's, ^{3}H, ^{3}He,...) interacting with *field electrons*, $x^{t/f}$ is usually much less than 1, indicating that collective contributions can be ignored [see $\ln\Lambda_c^{\alpha/e}$ in Fig. 1(a)]. In contrast, for charged-fusion products interacting with *field ions*, $x^{t/f}$ is usually much larger than 1, and therefore collective effects are significant [dashed line, Fig. 1(b)].

In order to illustrate the results of the generalized stopping power [Eq. (3)], we consider four cases: α's, ^{3}H, ^{3}He, and hot electrons each interacting with field ions and field electrons. For 3.5 MeV α's in a 10^{26}/cm^3 D-T plasma, Figs. 1(a) and 1(b) show the corresponding Coulomb logarithms for α-electron ($\ln\Lambda^{\alpha/e}$) and α-ion

TABLE I. The relative importance of 3.5 MeV α stopping by deuterons and tritons compared to that by electrons ($n_e = 10^{26}$/cm^3, $T_e \simeq T_i$).

T_e (keV)	D-T ion stopping (% of total)	Electron stopping (% of total)
1.0	≃6	≃94
5.0	≃19	≃81
10.0	≃32	≃68
20.0	≃47	≃53
40.0	≃64	≃36

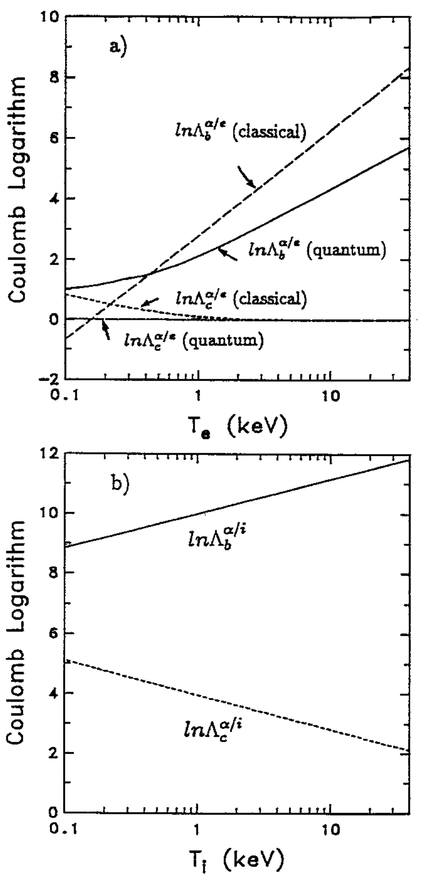

FIG. 1. (a) The Coulomb logarithms for α-electron interactions, for 3.5 MeV α's originating from D-T fusion ($n_e = 10^{26}$/cm^3). The quantum calculation (solid line) is used in our text and subsequent figures. The classical (Spitzer) calculation (long-dashed straight line) is given for reference. The effect of collective plasma oscillations for this particular case is unimportant. Stopping power and ρR are calculated only for $T_e \gtrsim 1$ keV, i.e., for $\ln\Lambda_b \gtrsim 2$. (For $\ln\Lambda_b^{\alpha/e} < 2$, strongly coupled effects become increasingly an issue [15,20].) (b) The Coulomb logarithm for α-ion (deuteron and triton) interactions ($\ln\Lambda_b^{\alpha/i}$) and α-ion collective interactions ($\ln\Lambda_c^{\alpha/i}$). In contrast to the α-electron interaction (a), quantum effects are unimportant. However, collective effects are significant since $v_\alpha \gg v_i$ [v_i, the background ion (D or T) velocity].

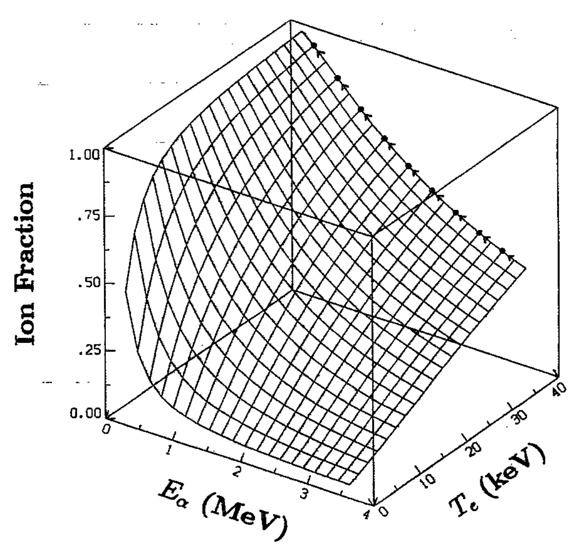

FIG. 2. The relative fraction of α-field ion (D-T) stopping as a function of the α kinetic energy (E_α) and plasma temperature ($T_e \sim T_i$). $n_e = 10^{26}$ cm^{-3}, although the ion fraction has only a very weak density dependence. For $E_\alpha \lesssim 1.5$ MeV and $T_e \gtrsim 15$ keV, ion stopping is dominant. A 3.5 MeV α in a 40 keV plasma deposits its energy along the dotted trajectory. It initially deposits ~35% of its energy to ions, but by the end of its range $\gtrsim$95% is going into the ions. By integrating over the trajectory, the total ion stopping is ≃64% (see Table I). This effect significantly reduces the α range (R) and ρR by about 73% (see Fig. 3).

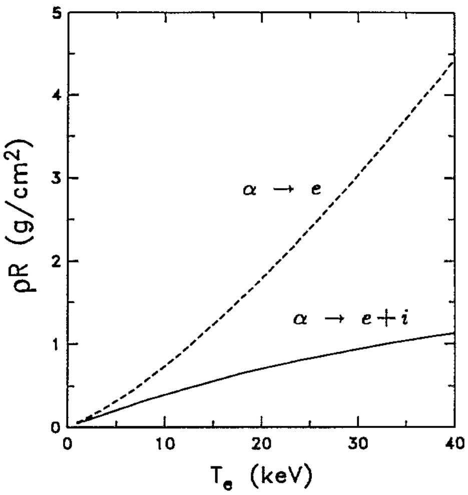

FIG. 3. ρR for 3.5 MeV α's interacting in a $10^{26}/\mathrm{cm}^3$ D-T plasma. The dashed line represents pure electron stopping (electron scattering is negligible). The solid line results from the cumulative effects of electron binary, ion binary (small-angle plus scattering), and ion collective oscillations.

($\ln\Lambda^{\alpha/i}$) interactions. In the case of α-electron interactions, Eqs. (1) and (2) nearly reduce to Trubnikov's results [8] because the mass ratio of field to test particles, m_e/m_α, is of order 10^{-4}. However, when the field-to-test mass ratio (m_f/m_t) is of order 1 or 10^3—as it is for α's, ^{3}H, and ^{3}He interacting with field ions, or for test electrons interacting with field ions—Eqs. (1), (2), and (3) must be used instead of Trubnikov's. Table I shows the relative importance of ion and electron stopping for α's that thermalize from 3.5 MeV. Note that ion stopping becomes significant for $T_e \sim T_i \gtrsim 5$ keV. In more detail, Fig. 2 plots the ion stopping fraction, $(dE^{\alpha/i}/dx)/(dE^{\alpha/i}/dx + dE^{\alpha/e}/dx)$, for relevant α energies (≤ 3.5 MeV) and plasma temperatures. Figure 3 shows the corresponding ρR for α's (calculated from the 3.5 MeV birth energy to background thermal temperature). For example, at 20 keV inclusion of ion stopping (binary plus collective) reduces the ρR of pure electron stopping by about 60%. Also for the α's, Fig. 4 shows the density dependence of ρR. Effects of electron degeneracy can be clearly seen for density $\gtrsim 10^{27}/\mathrm{cm}^3$ and temperature $\lesssim 5$ keV. Degeneracy effects enter in both the calculation of $\ln\Lambda$ and the parameter $x^{t/f}$. (In the degenerate regime, our calculations are only semiquantitative.) In the nondegenerate regime of Fig. 4, the results of Fraley *et al.* [14], which ignored (large angle) scattering and collective effects, are about 20% larger. (They did not treat the degenerate regime.)

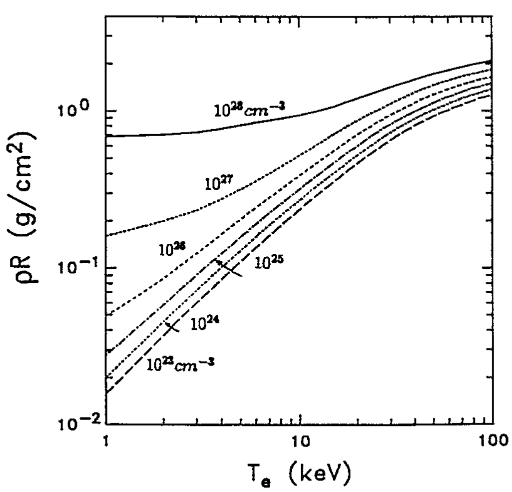

FIG. 4. ρR curves for 3.5 MeV α's interacting with D-T plasmas of various densities. Quantum degeneracy is important for $n_e \gtrsim 10^{27}/\mathrm{cm}^3$ and $T_e \lesssim 5$ keV.

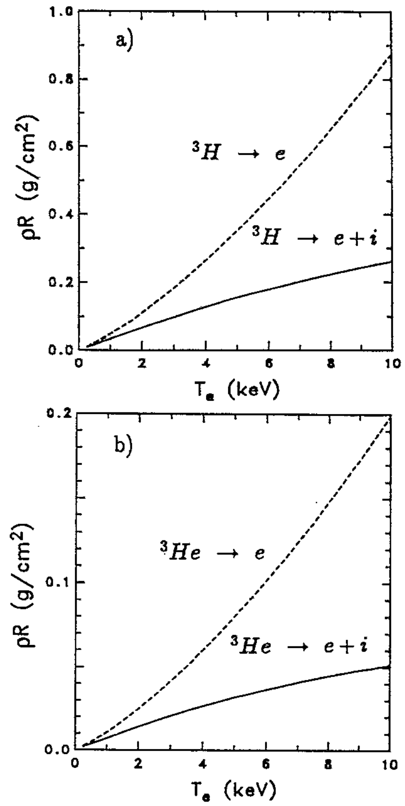

FIG. 5. (a),(b) ρR for 1.01 MeV ^{3}H and 0.82 MeV ^{3}He, respectively, interacting in a $6\times10^{24}/\mathrm{cm}^3$ D plasma (density of Ref. [4]). The dashed line represents pure electron stopping (electron scattering is negligible). The solid line results from the cumulative effects of electron binary, ion binary (small-angle plus scattering), and ion collective oscillations.

The development of novel ρR diagnostics is currently based upon the 1.01 MeV ^{3}H and 0.82 MeV ^{3}He [4,18] that result from D-D fusion. Because of the relevance of this diagnostic to present experiments, we show in Figs. 5(a) and 5(b) ρR with and without the effect of ion stopping. As is evident, even for fairly low plasma temperatures, the effects of ion stopping are extremely important.

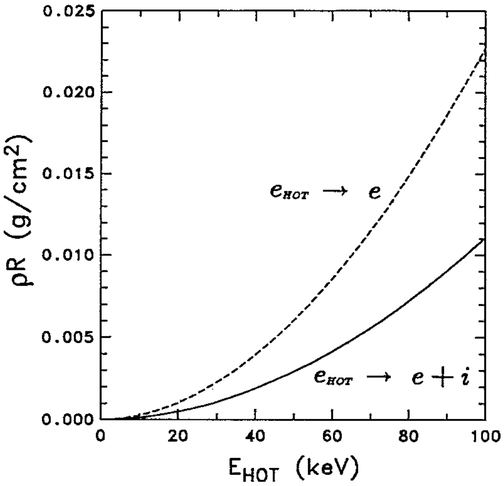

FIG. 6. ρR for hot corona electrons interacting with a cold (core) D plasma ($n_e = 10^{23}/\mathrm{cm}^3$ and $T_e = 50$ eV). The dashed curve shows the effects of pure small-angle binary collisions with electrons. The solid line shows, in addition, the cumulative effects of electron small-angle collisions with ions, large-angle scattering off electrons and ions, and electron oscillations.

In contrast to the charged-fusion products interacting with background electrons and ions, for which the scattering is small either because $m_e/m_\alpha \sim 10^{-4}$ or because $\ln\Lambda_b^{e/i} \sim 10$, scattering must be included in treating hot electrons interacting with cold electrons and ions. Such a situation arises when hot corona electrons interact with the cold core [19]. The dashed line in Fig. 6 shows ρR due only to small-angle binary collisions with electrons, which is the conventional calculation. The solid line includes as well large-angle scattering of electrons and ions plus the collective effects of the background electrons. As can be seen, these contributions are important.

In summary, we have calculated the stopping powers and ρR of charged-fusion products and hot electrons interacting with plasmas relevant to inertial confinement fusion. For the first time the effects of scattering, which limited previous calculations to upper limits [13,14], have been properly treated. In contrast to earlier work utilizing the binary collision approximation, these new calculations are justified for application to $\ln\Lambda_b \gtrsim 2$ plasmas. Furthermore, the important effects of ion stopping, electron quantum properties, and collective plasma oscillations have been treated within a unified framework. Ion stopping is found to be important for all charged-fusion products. For hot electrons interacting with cold dense plasmas, the contributions of scattering and collective oscillations are significant.

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